Math 102

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October 4, 2018

Announcements

- Thanksgiving on Monday October 8
  - We will have class Tuesday October 9
  - No pre-lecture WW
- Midterm signups - see email from Elyse Yeager

Goals Today

- Conceptual questions about $f(x), f'(x), f''(x)$
- More graph sketching - rational functions

Reminder: Terminology

- $f'(x)$ measures the slope of a function. When $f'(x) = 0$, it is called a critical point.

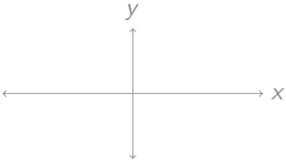
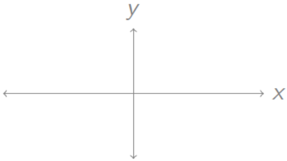
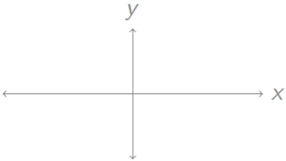
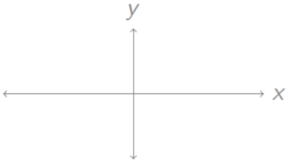
Reminder: Terminology

- $f'(x)$ measures the slope of a function. When $f'(x) = 0$, it is called a critical point.
- $f''(x)$ measures the concavity of a function.

$f''(x) > 0 \implies$ concave up
'like a cup'

$f''(x) < 0 \implies$ concave down
'like a frown'

Fill the table!

	concave up	concave down
increasing		
decreasing		

	concave up	concave down
increasing		
decreasing		

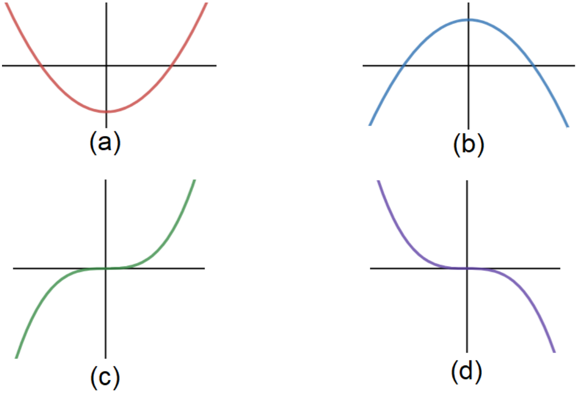
$f'(x)$ is...

(I) Negative when $x < 0$, negative when $x > 0$

(II) Negative when $x < 0$, positive when $x > 0$

(III) Positive when $x < 0$, negative when $x > 0$

(IV) Positive when $x < 0$, positive when $x > 0$



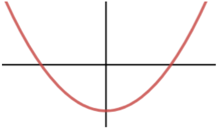
$f'(x)$ is...

(I) Negative when $x < 0$, negative when $x > 0$ (d)

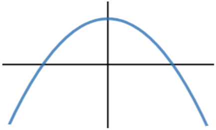
(II) Negative when $x < 0$, positive when $x > 0$ (a)

(III) Positive when $x < 0$, negative when $x > 0$ (b)

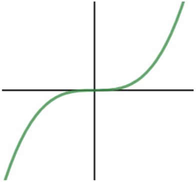
(IV) Positive when $x < 0$, positive when $x > 0$ (c)



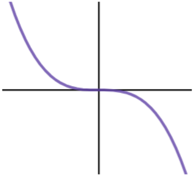
(a)



(b)



(c)



(d)

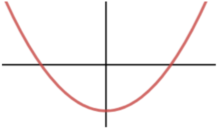
$f''(x)$ is...

(I) Negative for all x

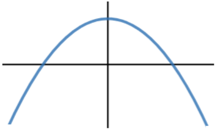
(II) Negative when $x < 0$, positive when $x > 0$

(III) Positive when $x < 0$, negative when $x > 0$

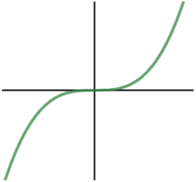
(IV) Positive for all x



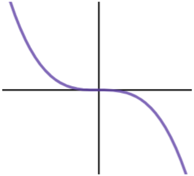
(a)



(b)



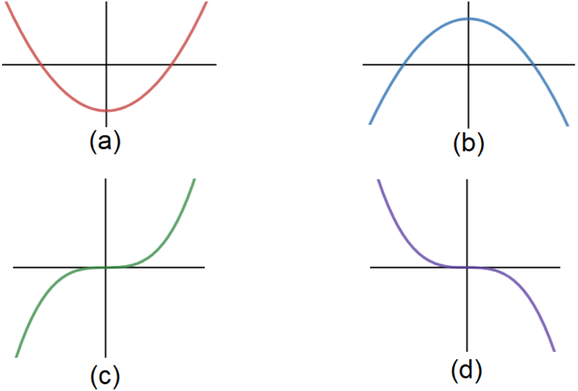
(c)



(d)

$f''(x)$ is...

(I) Negative for all x (b)
(II) Negative when $x < 0$, positive when $x > 0$ (c)
(III) Positive when $x < 0$, negative when $x > 0$ (d)
(IV) Positive for all x (a)



Classifying Critical Points

How do we determine if a critical point of $f(x)$ is a local minimum, a local maximum, or neither? There are two ways:

- We can look at $f'(x)$, the slope.
  - If $f'(x)$ changes from positive to negative, it's a local maximum.
  - If $f'(x)$ changes from negative to positive, it's a local minimum.
  - Otherwise, it is neither.
- We can test $f''(x)$, the concavity at the critical point.
  - If $f''(x) > 0$, it's a local minimum.
  - If $f''(x) < 0$, it's a local maximum.
  - If $f''(x) = 0$, it's more complicated...

Graph Sketching

Question: For the function

$$f(x) = \frac{1}{4}x^4 - 2x^3 + \frac{9}{2}x^2 + 1$$

- Find all critical points.
- Which are local maxima? Local minima? Neither?

Graph Sketching

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Use this information to sketch the function. Then label any inflection points.

Graph Sketching

$$f(x) = \frac{x^2}{x+1}$$

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- $f'(x) = \frac{x^2+2x}{(x+1)^2}$. Critical points at $x = 0, -2$.

Graph Sketching

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- Vertical asymptote at $x = -1$, zero at $x = 0$. Goes to $\pm\infty$ as $x \to \pm\infty$.
- $f'(x) = \frac{x^2+2x}{(x+1)^2}$. Critical points at $x = 0, -2$.
- By testing a point in each region, $f'(x)$ is positive for $x > 0$, negative for $-1 < x < 0$ or $-2 < x < -1$, and positive for $x < -2$.

Graph Sketching

$f(x) = \frac{x^2}{x+1}$

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- $f'(x) = \frac{x^2+2x}{(x+1)^2}$. Critical points at $x = 0, -2$.
- By testing a point in each region, $f'(x)$ is positive for $x > 0$, negative for $-1 < x < 0$ or $-2 < x < -1$, and positive for $x < -2$.
- $f''(x) = \frac{2}{(x+1)^3}$. This is positive for $x > -1$ and negative for $x < -1$.

Sketching Game

1. $f(x) = \frac{1}{4}x^4 - \frac{2}{3}x^3 + 1$
2. $f(x) = \frac{1}{4}x^4 - \frac{4}{3}x^3 + 2x^2 + 1$
3. $f(x) = \frac{x^2}{4} - \frac{2}{x}$
4. $f(x) = \frac{5(x-1)^2}{x^3}$

Recap

- Conceptual questions about $f(x), f'(x), f''(x)$
- More graph sketching - rational functions

Extra Practice

Consider the Hill function we discussed last class

$$P(x) = \frac{30x^3}{20^3 + x^3}$$

We found that the graph of $G(x) = 0.5x$ intersects $P(x)$ at two distinct positive values of x. On the other hand, when a is very large, $f(x) = ax$ does not intersect the graph of $P(x)$ at all! We claim that $f(x) = \frac{x}{\sqrt[3]{2}}$ is *tangent* to the graph of $P(x)$. Show that this is true.